

MST124

Specimen Examination Paper

Solutions

Correct option summary

The correct options are as follows.

Section A

Question	1	2	3	4	5	6	7	8	9	10
Correct option	C	A	E	A	A	B	C	D	D	C

Question	11	12	13	14	15	16	17	18	19	20
Correct option	A	D	B	D	D	A	D	B	E	B

Question	21	22	23	24	25	26	27	28	29	30
Correct option	D	E	C	B	E	C	C	B	E	C

Question	31	32	33	34
Correct option	C	E	A	B

Section B

Question	35	36	37	38	39	40	41	42
Correct option	D	B	B	D	C	A	C	D

Section A

Question 1

$$\begin{aligned}3(2x + 1) + (3 + x)(x - 2) &= 6x + 3 + 3x - 6 + x^2 - 2x \\ &= x^2 + 7x - 3\end{aligned}$$

References:

Unit 1, Examples 7 and 9 (pages 34 and 36)

The correct option is C.

Question 2

To clear the fractions, multiply every term on both sides of the equation by the lowest common multiple of the denominators, which is $6x$.

$$\begin{aligned}\frac{2}{x} &= \frac{3}{2x} + \frac{5}{3} \\ \frac{2 \times 6x}{x} &= \frac{3 \times 6x}{2x} + \frac{5 \times 6x}{3} \quad (\text{assuming } x \neq 0) \\ \frac{12x}{x} &= \frac{18x}{2x} + \frac{30x}{3} \\ 12 &= 9 + 10x \\ 3 &= 10x \\ \frac{3}{10} &= x\end{aligned}$$

The solution is $x = \frac{3}{10}$.

References:

Handbook, pages 17 and 18

Unit 1, Example 23 (page 83)

The correct option is A.

Question 3

Use the index laws summarised on page 6 of the Handbook.

$$\begin{aligned}a^{-1} b^2 \sqrt[3]{a^2 b^5} &= a^{-1} b^2 (a^2 b^5)^{1/3} \\ &= a^{-1} b^2 a^{2/3} b^{5/3} \\ &= a^{(-1+2/3)} b^{(2+5/3)} \\ &= a^{-1/3} b^{11/3} \\ &= \frac{1}{a^{1/3}} b^{11/3} \\ &= \frac{b^{11/3}}{a^{1/3}}\end{aligned}$$

References:

Handbook, pages 6 and 16

Unit 1, Examples 20 and 21 (pages 68 and 71)

The correct option is E.

Question 4

Following the strategy on page 18 of the Handbook, square both sides and clear the fraction. There is more than one term containing the required subject x , so collect all these terms on one side and then take x out as a common factor. Lastly, divide by the expression inside the brackets.

$$y = \sqrt{\frac{2x+z}{x}}$$

$$y^2 = \frac{2x+z}{x}$$

$$xy^2 = 2x + z$$

$$xy^2 - 2x = z$$

$$x(y^2 - 2) = z$$

$$x = \frac{z}{y^2 - 2} \quad (\text{assuming } y^2 \neq 2)$$

References:

Handbook, page 18

Unit 1, Example 24 (page 88)

The correct option is A.

Question 5

The gradient of the straight line passing through the points $(5, 2)$ and $(7, -3)$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{7 - 5} = -\frac{5}{2}.$$

References:

Handbook, page 19

Unit 2, Activity 4 (page 128)

The correct option is A.

Question 6

Completing the square gives

$$\begin{aligned} 3x^2 - 12x + 5 &= 3(x^2 - 4x) + 5 \\ &= 3((x - 2)^2 - 4) + 5 \\ &= 3(x - 2)^2 - 12 + 5 = 3(x - 2)^2 - 7. \end{aligned}$$

So the value of s is -7 .

References:

Handbook, page 21

Unit 2, Example 13 (page 168)

The correct option is B.

Question 7

The coefficient of x^2 is positive, so the graph is u-shaped.

Completing the square in the right-hand side gives

$$\begin{aligned}y &= x^2 + 4x + 3 \\&= (x + 2)^2 - 4 + 3 \\&= (x + 2)^2 - 1.\end{aligned}$$

Therefore, the vertex of the graph with the equation $y = x^2 + 4x + 3$ is $(-2, -1)$.

The vertex of graph **C** is in the correct quadrant.

References:

Handbook, pages 21 and 23

Unit 2, Example 20 (page 180)

The completed-square form also shows that the correct graph is obtained by translating the graph of $y = x^2$ to the left by 2 units and down by 1 unit, which leads to the same conclusion.

References:

Handbook, page 28

Unit 3, Example 3 (page 242)

Alternatively, putting $y = 0$ and factorising the right-hand side gives

$$\begin{aligned}x^2 + 4x + 3 &= 0 \\(x + 1)(x + 3) &= 0 \\x + 1 = 0 \quad \text{or} \quad x + 3 &= 0 \\x = -1 \quad \text{or} \quad x &= -3.\end{aligned}$$

So the x -intercepts are -1 and -3 .

Again, this identifies graph **C** as the correct graph.

References:

Handbook, pages 21 and 22

Unit 2, Example 19 (page 179)

The correct option is C.

Question 8

To find the x -intercept, put $y = 0$, which gives

$$3x + 4 \times 0 = 6.$$

Solving this equation gives

$$\begin{aligned}3x &= 6 \\x &= 2.\end{aligned}$$

So the x -intercept is 2.

References:

Handbook, page 19

Unit 2, Example 3 (page 132)

The correct option is D.

Question 9

A function is one-to-one if each output value is obtained from only one input value. So any horizontal line intersects the graph of a one-to-one function no more than once. Therefore only graph **D** is the graph of a one-to-one function.

References:

Handbook, page 26

Unit 3, Activity 35 (page 257)

The correct option is D.

Question 10

Pages 13 and 14 of the Handbook are a useful reference for notation such as this.

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 + 2 = x + 2$$

References:

Handbook, page 26

Unit 3, Example 4 (page 252)

The correct option is C.

Question 11

The equation $f(x) = y$ gives

$$5x + 2 = y.$$

Rearranging to express x in terms of y gives

$$5x = y - 2$$

$$x = \frac{y - 2}{5}.$$

Hence the rule of f^{-1} is $f^{-1}(y) = \frac{y-2}{5}$;

that is,

$$f^{-1}(x) = \frac{x - 2}{5}.$$

References:

Handbook, page 26

Unit 3, Example 6 (page 258)

The correct option is A.

Question 12

Use the logarithm laws summarised on page 6 of the Handbook.

$$\ln(x^2 e^{4y}) = \ln x^2 + \ln e^{4y} = 2 \ln x + 4y$$

References:

Handbook, page 6

Unit 3, Activities 45 and 46 (pages 277 and 279)

The correct option is D.

Question 13

Rearranging the inequality $x^2 \leq 12 + x$ gives

$$x^2 - x - 12 \leq 0.$$

The graph of $f(x) = x^2 - x - 12$ is u-shaped. Its x -intercepts are given by

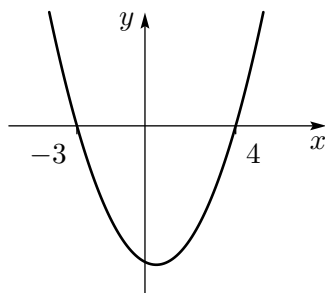
$$x^2 - x - 12 = 0;$$

that is

$$(x + 3)(x - 4) = 0,$$

so they are -3 and 4 .

The graph shown below lies on or below the x -axis when $-3 \leq x \leq 4$, so $x^2 - x - 12$ is less than or equal to zero when $-3 \leq x \leq 4$. Hence the solution set is $[-3, 4]$.



References:

Handbook, pages 24 and 32

Unit 3, Example 13 (page 295)

Alternatively, you can use a table of signs. The inequality can be written as

$$x^2 - x - 12 \leq 0,$$

which can be factorised as

$$(x + 3)(x - 4) \leq 0.$$

A factor is equal to 0 when $x = -3$ and $x = 4$.

A table of signs for the expression on the left-hand side of the inequality is given below.

x	$(-\infty, -3)$	-3	$(-3, 4)$	4	$(4, \infty)$
$x + 3$	$-$	0	$+$	$+$	$+$
$x - 4$	$-$	$-$	$-$	0	$+$
$(x + 3)(x - 4)$	$+$	0	$-$	0	$+$

The expression $(x + 3)(x - 4)$ is negative or zero when $-3 \leq x \leq 4$ so, as before, the solution set is $[-3, 4]$.

References:

Handbook, page 32

Unit 3, Example 14 (page 297)

The correct option is B.

Question 14

Questions 14, 15 and 16 involve finding unknown lengths and angles in triangles. The decision tree on page 37 of the Handbook can help you to decide which method to use.

Here we have a right-angled triangle with a given angle, so we can use 'SOH CAH TOA'. This is summarised on page 4 of the Handbook.

Let x be the length of the hypotenuse. The labelled side length is opp = 6, so

$$\sin\left(\frac{\pi}{5}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{6}{x}.$$

Hence

$$x = \frac{6}{\sin(\pi/5)} = 10.207\dots$$

The length of the hypotenuse is 10.2 (to 1 d.p.).

References:

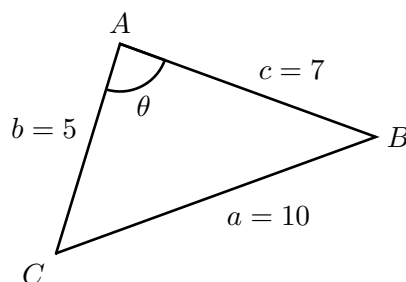
Handbook, pages 4, 33 and 37

Unit 4, Example 2 (page 15)

The correct option is D.

Question 15

The triangle in this question doesn't have a right angle, so choose between the sine rule and the cosine rule. You don't know a 'side length – opposite angle' pair, so use the cosine rule. First it helps to label the angles and sides of the triangle.



Rearranging the cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ gives

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

So

$$\cos \theta = \frac{5^2 + 7^2 - 10^2}{2 \times 5 \times 7} = -\frac{26}{70} = -\frac{13}{35}.$$

Therefore

$$\theta = \cos^{-1}\left(-\frac{13}{35}\right) = 111.803\dots^\circ = 112^\circ \text{ (to the nearest degree).}$$

References:

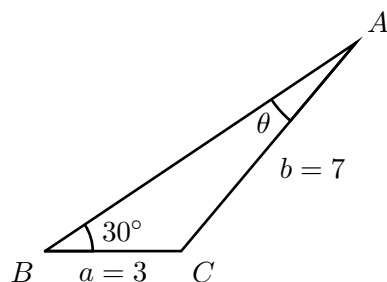
Handbook, page 37

Unit 4, Example 12 (page 64)

The correct option is D.

Question 16

Again it helps to label the sides and angles. This triangle also doesn't have a right angle, but this time you know side b and opposite angle B , so use the sine rule. Remember that when you use the sine rule to find an angle you obtain two possible solutions. Sometimes it is possible to determine which possibility is correct, and sometimes it is not.



By the sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

so

$$\frac{3}{\sin \theta} = \frac{7}{\sin 30^\circ}.$$

Hence

$$\sin \theta = \frac{3 \sin 30^\circ}{7} = \frac{3 \times 1/2}{7} = \frac{3}{14}.$$

Therefore,

$$\theta = \sin^{-1}\left(\frac{3}{14}\right) = 12.373\dots^\circ$$

or

$$\theta = 180^\circ - 12.373\dots^\circ = 167.626\dots^\circ.$$

The obtuse angle $\theta = 167.626\dots^\circ$ is impossible in this case because

$$167.626\dots^\circ + 30^\circ > 180^\circ.$$

Therefore $\theta = 12^\circ$, to the nearest degree.

References:

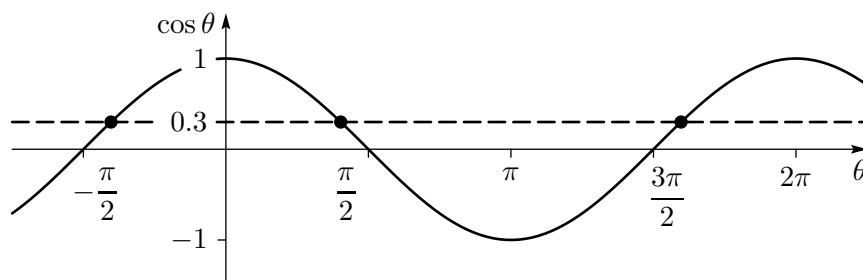
Handbook, page 37

Unit 4, Example 10 (page 59)

The correct option is A.

Question 17

Useful graphs are given on page 9 of the Handbook, including the graph of $y = \cos \theta$.



The line $y = 0.3$ intersects the graph of $y = \cos \theta$ three times in the interval $[-\pi/2, 2\pi]$. Therefore, there are three solutions to the equation $\cos \theta = 0.3$. These are the θ -coordinates of the crossing points.

References:

Handbook, page 9

Unit 4, Example 8 (page 51)

The correct option is D.

Question 18

The circle with centre (a, b) and radius r has equation

$$(x - a)^2 + (y - b)^2 = r^2.$$

Therefore, the equation of the circle with centre $(3, -2)$ and radius 4 is

$$(x - 3)^2 + (y - (-2))^2 = 4^2;$$

that is

$$(x - 3)^2 + (y + 2)^2 = 16.$$

Expanding the brackets gives

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 16,$$

and simplifying gives

$$x^2 + y^2 - 6x + 4y - 3 = 0.$$

References:

Handbook, page 38

Unit 5, Example 3 and page 113

The correct option is B.

Question 19

The component form of a two-dimensional vector $\mathbf{v}$ that makes an angle θ with the positive x -direction is given by the formula

$$\mathbf{v} = |\mathbf{v}| \cos \theta \mathbf{i} + |\mathbf{v}| \sin \theta \mathbf{j}.$$

Therefore, a vector with magnitude 4 that makes an angle of 120° with the positive x -direction has component form

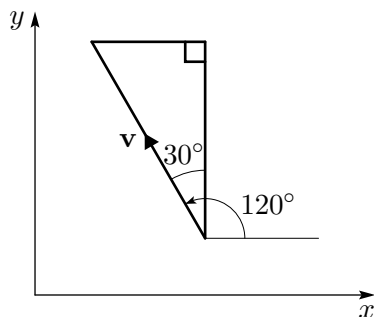
$$\begin{aligned}\mathbf{v} &= 4 \cos 120^\circ \mathbf{i} + 4 \sin 120^\circ \mathbf{j} \\ &= -4 \cos 60^\circ \mathbf{i} + 4 \sin 60^\circ \mathbf{j} \\ &= -4 \times \frac{1}{2} \mathbf{i} + 4 \times \frac{\sqrt{3}}{2} \mathbf{j} \\ &= -2\mathbf{i} + 2\sqrt{3}\mathbf{j}.\end{aligned}$$

References:

Handbook, page 42

Unit 5, Activity 48 (page 179)

Alternatively, you can sketch the vector and a right-angled triangle. Use trigonometry to find the magnitudes of the components, and use the direction of the vector to find the signs of the components.



References:

Handbook, page 42

Unit 5, Example 19 (page 176)

The correct option is E.

Question 20

Use the index laws on page 6 of the Handbook to rewrite the function in the form ax^n before differentiating.

The function is $f(x) = \frac{1}{2x^{2/3}} = \frac{1}{2}x^{-2/3}$.

So

$$f'(x) = \frac{1}{2} \times \left(-\frac{2}{3}\right) x^{-5/3} = -\frac{1}{3} \times \frac{1}{x^{5/3}} = -\frac{1}{3x^{5/3}}.$$

References:

Handbook, pages 6, 7 and 44

Unit 6, Example 3 (page 242)

The correct option is B.

Question 21

$g(x) = 10x^4 - 3x^2 + 8$, so, by the sum rule and constant multiple rule for derivatives,

$$\begin{aligned} g'(x) &= 10 \times 4x^3 - 3 \times 2x \\ &= 40x^3 - 6x. \end{aligned}$$

References:

Handbook, pages 7 and 44

Unit 6, Example 4 (page 247)

The correct option is D.

Question 22

Page 49 of the Handbook helps in choosing which method to use when differentiating.

The function $p(x) = x^3 \ln x$ is of the form ‘something $\times$ something’ so use the product rule.

Remember the table of standard derivatives on page 7 of the Handbook to help with differentiating things like $\ln x$.

$p(x) = x^3 \ln x$, so, by the product rule,

$$\begin{aligned} p'(x) &= x^3 \times \frac{1}{x} + \ln x \times 3x^2 \\ &= 3x^2 \ln x + x^2. \end{aligned}$$

References:

Handbook, pages 7, 48 and 49

Unit 7, Example 2 (page 16)

The correct option is E.

Question 23

The function $r(t) = \tan(t^2 - 2t)$ is of the form ‘a function of something’ where the ‘something’ is the $t^2 - 2t$ inside the brackets, so use the chain rule.

Remember the table of standard derivatives on page 7 of the Handbook to help with differentiating things like $\tan$.

$r(t) = \tan(t^2 - 2t)$, so, by the chain rule,

$$\begin{aligned} r'(t) &= \sec^2(t^2 - 2t) \times (2t - 2) \\ &= 2(t - 1) \sec^2(t^2 - 2t). \end{aligned}$$

References:

Handbook, pages 7, 48 and 49

Unit 7, Example 8 (page 28)

The correct option is C.

Question 24

You can refer to the table of standard indefinite integrals on page 7 of the Handbook. You can also use the sum and constant multiple rules for indefinite integrals if you find them helpful.

$$\begin{aligned}\int (x^{3/2} + 2 \sin x) \, dx &= \int x^{3/2} \, dx + 2 \int \sin x \, dx \\ &= \frac{1}{5/2} x^{5/2} - 2 \cos x + c \\ &= \frac{2}{5} x^{5/2} - 2 \cos x + c\end{aligned}$$

References:

Handbook, pages 7 and 52

Unit 8, Example 11 (page 154)

The correct option is B.

Question 25

$$\int_2^5 f(x) \, dx = \int_2^3 f(x) \, dx + \int_3^5 f(x) \, dx$$

so

$$4 = -2 + \int_3^5 f(x) \, dx$$

and hence

$$\int_3^5 f(x) \, dx = 4 - (-2) = 6.$$

References:

Handbook, page 51

Unit 8, Activity 7 (page 121)

The correct option is E.

Question 26

When asked to find the area between a graph and the x -axis, use definite integrals. You need to be careful though whether the graph lies above or below the x -axis in the interval of interest. Signed areas below the x -axis are negative so need to be evaluated separately and the minus sign removed.

Here, $\sin x > 0$ when $x \in [0, \pi/2]$ and $x > 0$ when $x \in [0, \pi/2]$. Hence $x + \sin x > 0$ when $x \in [0, \pi/2]$. Therefore, the graph of $y = x + \sin x$ lies above the x -axis for values of x in the interval $[0, \pi/2]$. The area between this graph and the x -axis from $x = 0$ to $x = \pi/2$ is therefore

$$\begin{aligned} \int_0^{\pi/2} (x + \sin x) \, dx &= \left[\frac{1}{2}x^2 - \cos x \right]_0^{\pi/2} \\ &= \frac{1}{2} \left[x^2 \right]_0^{\pi/2} - [\cos x]_0^{\pi/2} \\ &= \frac{1}{2} \left(\left(\frac{\pi}{2} \right)^2 - 0^2 \right) - \left(\cos \left(\frac{\pi}{2} \right) - \cos 0 \right) \\ &= \frac{1}{2} \times \frac{\pi^2}{4} - (0 - 1) \\ &= \frac{\pi^2}{8} + 1. \end{aligned}$$

References:

Handbook, pages 7 and 50–2

Unit 8, Examples 4 and 6 (pages 135 and 139)

The correct option is C.

Question 27

You could use integration by substitution here but it's worth looking out for a function of a linear expression. Page 52 of the *Handbook* gives a simple rule for integrating this. Here the linear expression is $4x - 1$.

$$\int e^{4x-1} \, dx = \frac{1}{4}e^{4x-1} + c$$

References:

Handbook, pages 7 and 52

Unit 8, Activity 30 (page 170)

The correct option is C.

Question 28

$$\begin{aligned} 3\mathbf{P} - 2\mathbf{Q} &= 3 \begin{pmatrix} 4 & 2 \\ 1 & 6 \end{pmatrix} - 2 \begin{pmatrix} 3 & 1 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 12 & 6 \\ 3 & 18 \end{pmatrix} - \begin{pmatrix} 6 & 2 \\ 2 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 6 & 4 \\ 1 & 18 \end{pmatrix} \end{aligned}$$

References:

Handbook, page 54

Unit 9, Activity 5 (page 227)

The correct option is B.

Question 29

You can save time here by working out only some of the elements of the answer matrix. You might work out enough elements to identify the correct answer, and one or two more as a check.

$$\begin{aligned}
 \mathbf{RS} &= \begin{pmatrix} 3 & 2 & 1 \\ 0 & 1 & 3 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 4 & 2 \\ 2 & 0 & 1 \\ 0 & 3 & 2 \end{pmatrix} \\
 &= \begin{pmatrix} 3 \times 1 + 2 \times 2 + 1 \times 0 & 3 \times 4 + 2 \times 0 + 1 \times 3 & 3 \times 2 + 2 \times 1 + 1 \times 2 \\ 0 \times 1 + 1 \times 2 + 3 \times 0 & 0 \times 4 + 1 \times 0 + 3 \times 3 & 0 \times 2 + 1 \times 1 + 3 \times 2 \\ 4 \times 1 + 2 \times 2 + 0 \times 0 & 4 \times 4 + 2 \times 0 + 0 \times 3 & 4 \times 2 + 2 \times 1 + 0 \times 2 \end{pmatrix} \\
 &= \begin{pmatrix} 7 & 15 & 10 \\ 2 & 9 & 7 \\ 8 & 16 & 10 \end{pmatrix}
 \end{aligned}$$

In the exam it may be quicker and easier to work out the individual elements in your head.

References:

Handbook, page 55

Unit 9, Example 1 (page 231)

The correct option is E.

Question 30

$$\det \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix} = 2 \times (-5) - (-3) \times 4 = -10 - (-12) = 2$$

References:

Handbook, page 56

Unit 9, page 255

The correct option is C.

Question 31

The given recurrence system specifies a geometric sequence with first term 10 and common ratio $\frac{1}{4}$. The closed form of a geometric sequence with first term a and common ratio r is

$$x_n = ar^{n-1} \quad (n = 1, 2, 3, \dots).$$

Therefore, the closed form of the given sequence is

$$x_n = 10 \left(\frac{1}{4}\right)^{n-1} \quad (n = 1, 2, 3, \dots).$$

Note that the range of values for n begins with 1 for the closed form but 2 for the recurrence system.

References:

Handbook, page 59

Unit 10, Example 7 (page 29)

The correct option is C.

Question 32

Use the table describing the long-term behaviour of the sequence (r^n) , together with the summarised 'Effects of multiplying each term by a constant' on page 60 of the Handbook.

Since $2 > 1$, the sequence (2^n) is increasing and $2^n \rightarrow \infty$ as $n \rightarrow \infty$.

The sequence (a_n) is obtained by multiplying each term of the sequence (2^n) by the by the negative constant $-\frac{1}{5}$. Hence the sequence (a_n) is decreasing and $a_n \rightarrow -\infty$ as $n \rightarrow \infty$. The only true statement is **E**.

References:

Handbook, page 60

Unit 10, Example 12 (page 46)

The correct option is E.

Question 33

To simplify a quotient of complex numbers, multiply the numerator and denominator by the complex conjugate of the denominator.

$$\begin{aligned}\frac{2+6i}{1-i} &= \frac{(2+6i)(1+i)}{(1-i)(1+i)} \\ &= \frac{2+2i+6i+6i^2}{1^2+(-1)^2} \\ &= \frac{2+8i-6}{2} \\ &= \frac{-4+8i}{2} \\ &= -2+4i\end{aligned}$$

References:

Handbook, page 65

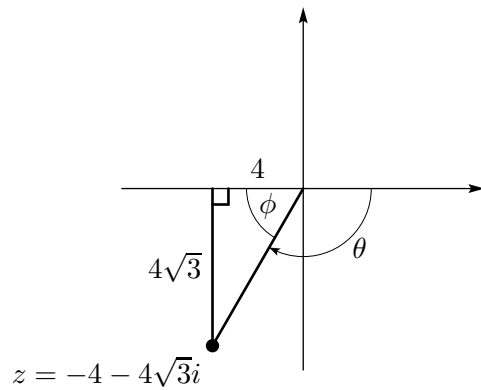
Unit 12, Example 3 (page 188)

The correct option is A.

Question 34

For this question you can use Step 2 of ‘To convert from Cartesian form to polar or exponential form’ on page 66 of the Handbook.

The trigonometric ratios for special angles are on page 4 of the Handbook.



From the diagram,

$$\tan \phi = \frac{4\sqrt{3}}{4} = \sqrt{3}.$$

Therefore $\phi = \pi/3$. So the principal argument is

$$\theta = -(\pi - \phi) = -\left(\pi - \frac{\pi}{3}\right) = -\frac{2\pi}{3}.$$

References:

Handbook, page 66

Unit 12, Example 5 (page 199)

The correct option is B.

Section B

Question 35

To find the points of intersection of any combination of lines, curves or circles, solve their equations simultaneously.

Using the equation of the line, $y = 2x + 1$, to substitute for y in the equation of the circle gives

$$x^2 + (2x + 1)^2 - 2x - 6(2x + 1) - 10 = 0.$$

Expanding brackets and simplifying gives

$$x^2 + 4x^2 + 4x + 1 - 2x - 12x - 6 - 10 = 0$$

$$5x^2 - 10x - 15 = 0$$

$$x^2 - 2x - 3 = 0.$$

This factorises as

$$(x + 1)(x - 3) = 0,$$

giving

$$x + 1 = 0 \quad \text{or} \quad x - 3 = 0,$$

so the solutions are $x = -1$ and $x = 3$.

Hence the x -coordinates of the points where the line intersects the circle are -1 and 3 .

Reference:

Unit 5, Example 8 (page 122)

The correct option is D.

Question 36

Notice that you've been given the derivative and that it has already been factorised. The multiple-choice options each have the same stationary points so it isn't really necessary to work these out, but it's quick to confirm them.

To find the stationary points, solve the equation $f'(x) = 0$, which gives

$$7(x+5)(x-3)^2 = 0,$$

that is,

$$x = -5 \quad \text{or} \quad x = 3.$$

To determine the nature of these stationary points, construct a table of signs using each factor of $f'(x)$.

x	$(-\infty, -5)$	-5	$(-5, 3)$	3	$(3, \infty)$
7	+	+	+	+	+
$x+5$	−	0	+	+	+
$(x-3)^2$	+	+	+	0	+
$f'(x)$	−	0	+	0	+
slope of f	$\searrow$	—	$\nearrow$	—	$\nearrow$

The stationary point at $x = -5$ is a local minimum and the stationary point at $x = 3$ is a horizontal point of inflection.

The constant factor 7 doesn't change the result but a negative constant factor would so should be included in the table of signs. Note also that the squared bracket is important (instead of the factor $(x-3)^2$, you could include the factor $x-3$ twice).

References:

Handbook, page 45

Unit 6, Example 8 (page 266)

Alternatively, you can choose sample points such as -6 , 0 and 4 . The values -6 and 0 lie on either side of the stationary point at $x = -5$, and the values 0 and 4 lie on either side of the stationary point at $x = 3$.

The function f is a polynomial function so is differentiable at all values of x . Also, there are no stationary points between -6 and -5 or between -5 and 0 . Similarly, there are no stationary points between 0 and 3 or between 3 and 4 .

Since $f'(x) = 7(x+5)(x-3)^2$, we have

$$f'(-6) = 7(-6+5)(-6-3)^2 = 7 \times (-1) \times (-9)^2 = -567$$

$$f'(0) = 7(0+5)(0-3)^2 = 7 \times 5 \times (-3)^2 = 315$$

$$f'(4) = 7(4+5)(4-3)^2 = 7 \times 9 \times 1^2 = 63.$$

Since the derivative is negative at $x = -6$ and positive at $x = 0$, the stationary point at $x = -5$ is a local minimum.

Since the derivative is positive at both $x = 0$ and $x = 4$, the stationary point at $x = 3$ is a horizontal point of inflection.

References:

Handbook, page 45

Unit 6, Example 9 (page 269)

The second derivative test is not a good approach here, since it can't identify points of inflection. Also, finding the second derivative would add a little more work, especially as the first derivative is given in factorised form.

The correct option is B.

Question 37

This is a function $\sin$ of a function $(2 - 3x)^2$ of x . So use the chain rule. Also, $(2 - 3x)^2$ is itself a function (squared) of $2 - 3x$, which is a function of x . So you need to use the chain rule multiple times.

$$\begin{aligned}\frac{d}{dx} \left(\sin \left((2 - 3x)^2 \right) \right) &= \cos \left((2 - 3x)^2 \right) \frac{d}{dx} \left((2 - 3x)^2 \right) && \text{by the chain rule} \\ &= \cos \left((2 - 3x)^2 \right) \times 2(2 - 3x) \frac{d}{dx} (2 - 3x) && \text{by the chain rule again} \\ &= \cos \left((2 - 3x)^2 \right) \times 2(2 - 3x) \times (-3) \\ &= -6(2 - 3x) \cos \left((2 - 3x)^2 \right)\end{aligned}$$

References:

Handbook, page 48

Unit 7, Example 12 (page 34)

The correct option is B.

Question 38

You've been given the velocity of an object and asked to find its displacement. You can look up 'displacement' in the index of the Handbook, which leads to page 47. Velocity and displacement are related by $v = ds/dt$, that is, velocity is the derivative of displacement with respect to time. So to find the displacement of an object given its velocity, you need to do the opposite: you need to integrate. The definitions

$$s = \int v \, dt \quad \text{and} \quad v = \frac{ds}{dt}$$

might be useful extra annotations for this page of your Handbook.

The displacement s (in m) of the object at time t (in seconds) is given by

$$\begin{aligned}s &= \int v \, dt = \int (1 - 6t^2) \, dt \\ &= t - 6 \times \frac{1}{3}t^3 + c \\ &= t - 2t^3 + c, \quad \text{where } c \text{ is an arbitrary constant.}\end{aligned}$$

It's important to remember the c here. This is why you've been given the displacement at some other time.

When $t = 0$, $s = 10$, so

$$10 = 0 - 2 \times 0^3 + c,$$

which gives $c = 10$.

So the equation for the displacement of the object in terms of time is

$$s = t - 2t^3 + 10.$$

When $t = 4$,

$$s = 4 - 2 \times 4^3 + 10 = -114.$$

So the displacement of the object at $t = 4$ is -114 m.

References:

Handbook, page 47

Unit 8, Example 19 (page 171)

The correct option is D.

Question 39

Page 53 of the Handbook helps in choosing which method to use when integrating.

In this case, the integrand is of the form '(something)² × the derivative of the something', so use integration by substitution.

Let $u = \cos(3x)$; then $\frac{du}{dx} = -3 \sin(3x)$.

Putting $x = 0$ in $u = \cos(3x)$ gives $u = \cos 0 = 1$.

Putting $x = \pi/9$ in $u = \cos(3x)$ gives $u = \cos \frac{\pi}{3} = \frac{1}{2}$.

So,

$$\begin{aligned} \int_0^{\pi/9} \sin(3x) \cos^2(3x) \, dx &= -\frac{1}{3} \int_0^{\pi/9} (-3 \sin(3x)) \cos^2(3x) \, dx \\ &= -\frac{1}{3} \int_1^{1/2} u^2 \, du \\ &= -\frac{1}{3} \left[\frac{u^3}{3} \right]_1^{1/2} \\ &= -\frac{1}{9} [u^3]_1^{1/2} \\ &= -\frac{1}{9} \left(\frac{1}{2}^3 - 1^3 \right) \\ &= -\frac{1}{9} \left(\frac{1}{8} - 1 \right) \\ &= -\frac{1}{9} \left(-\frac{7}{8} \right) \\ &= \frac{7}{72} \end{aligned}$$

References:

Handbook, page 52

Unit 8, Example 1 (page 173)

The correct option is C.

Question 40

Page 53 of the Handbook helps in choosing which method to use when integrating.

In this case the integrand is of the form ‘something $\times$ something’, that is $f(x)g(x)$. If you choose $f(x) = 3x + 1$, then f becomes simpler when differentiated. So use integration by parts.

Integrating by parts gives

$$\begin{aligned}\int (3x + 1)e^{2x} \, dx &= (3x + 1) \times \frac{1}{2}e^{2x} - \int 3 \times \frac{1}{2}e^{2x} \, dx \\&= \frac{1}{2}e^{2x}(3x + 1) - \frac{3}{2} \int e^{2x} \, dx \\&= \frac{1}{2}e^{2x}(3x + 1) - \frac{3}{2} \times \frac{1}{2}e^{2x} + c \\&= \frac{1}{2}e^{2x}(3x + 1) - \frac{3}{4}e^{2x} + c \\&= \frac{1}{4}e^{2x}(2(3x + 1)) - 3) + c \\&= \frac{1}{4}e^{2x}(6x + 2 - 3) + c \\&= \frac{1}{4}e^{2x}(6x - 1) + c.\end{aligned}$$

References:

Handbook, page 53

Unit 8, Example 24 (page 179)

The correct option is A.

Question 41

By the binomial theorem, each term in the expansion of $(a + b)^n$ is of the form

$${}^nC_k a^{n-k} b^k.$$

So with $n = 10$, $a = 2t$ and $b = 1/t^2$, each term in the expansion of $(2t + 1/t^2)^{10}$ is of the form

$${}^{10}C_k (2t)^{10-k} \left(\frac{1}{t^2}\right)^k.$$

Using index laws to collect powers of t together gives

$$\begin{aligned} {}^{10}C_k (2t)^{10-k} \left(\frac{1}{t^2}\right)^k &= {}^{10}C_k 2^{10-k} t^{10-k} (t^{-2})^k \\ &= {}^{10}C_k 2^{10-k} t^{10-k} t^{-2k} \\ &= {}^{10}C_k 2^{10-k} t^{10-3k}. \end{aligned}$$

The term in t^{-5} is obtained when $10 - 3k = -5$, which gives $-3k = -15$; that is, $k = 5$. Hence the coefficient of t^{-5} is

$${}^{10}C_5 2^{10-5} = 252 \times 2^5 = 8064.$$

References:

Handbook, page 61

Unit 10, Example 23 (page 81)

The correct option is C.

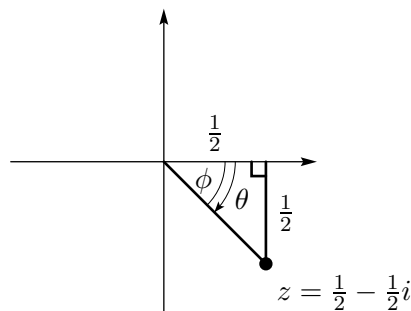
Question 42

As described on page 67 of the Handbook, to raise a complex number to a power you use de Moivre's formula. First you need to convert the complex number from Cartesian to polar form using the steps on page 66 of the Handbook. Convert back to Cartesian form at the end.

The modulus of $\frac{1}{2} - \frac{1}{2}i$ is

$$\sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}.$$

To find the principal argument, sketch $\frac{1}{2} - \frac{1}{2}i$ in the complex plane.



From the diagram,

$$\tan \phi = \frac{1/2}{1/2} = 1.$$

Therefore $\phi = \pi/4$. So the principal argument is $\theta = -\phi = -\frac{\pi}{4}$.

(Alternatively you may be able to see immediately from the diagram that the principal argument is $-\pi/4$.)

In polar form,

$$\frac{1}{2} - \frac{1}{2}i = \frac{1}{\sqrt{2}} \left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right).$$

Hence, by de Moivre's theorem,

$$\begin{aligned} \left(\frac{1}{2} - \frac{1}{2}i\right)^{-7} &= \left(\frac{1}{\sqrt{2}}\right)^{-7} \left(\cos\left(-7 \times \left(-\frac{\pi}{4}\right)\right) + i \sin\left(-7 \times \left(-\frac{\pi}{4}\right)\right) \right) \\ &= \left(\sqrt{2}\right)^7 \left(\cos\left(\frac{7\pi}{4}\right) + i \sin\left(\frac{7\pi}{4}\right) \right) \\ &= \left(\sqrt{2}\right)^7 \left(\cos\left(\frac{\pi}{4}\right) - i \sin\left(\frac{\pi}{4}\right) \right) \\ &= \left(\sqrt{2}\right)^7 \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right) \\ &= \left(\sqrt{2}\right)^6 (1 - i) \\ &= 2^3 (1 - i) \\ &= 8 - 8i. \end{aligned}$$

References:

Handbook, pages 66 and 67

Unit 12, Example 11 (page 212)

The correct option is D.

